

Bose-Fermi Equivalence in Three Dimensional Non-commutative Space-Time

Ajith K M*, E. Harikumar[†] and M. Sivakumar[‡]
School of Physics, University of Hyderabad,
Hyderabad, 500046, India.

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Abstract

We study the Fermionisation of Seiberg-Witten mapped action (to order θ) of the $\lambda\phi^4$ theory coupled minimally with U(1) gauge field governed by Chern-Simon action. Starting from the corresponding partition function we derive non-perturbatively (in coupling constant) the partition function of the spin $\frac{1}{2}$ theory following Polyakov spin factor formalism. We find the dual interacting fermionic theory is non local. This feature persists also in the limit of vanishing self coupling. In $\theta \rightarrow 0$ limit, the commutative result is regained.

*ph01ph13@uohyd.ernet.in

[†]harisp@uohyd.ernet.in

[‡]mssp@uohyd.ernet.in

1 Introduction

Non commutative (NC) space-time[1] has gained a lot of interest in recent time due to (i) relevance to quantum aspects of gravity (ii) as a regularization in field theory (iii) certain limit of the string theory. Well studied among them is the Moyal space-time whose co-ordinates obey

$$[x_\mu, x_\nu]_* = i\theta_{\mu\nu} \quad (1)$$

In Moyal space-time, the usual product is replaced by the $*$ product defined as

$$f(x) * g(x) = e^{\frac{i}{2}\theta^{ij}\partial_i^x\partial_j^y} f(x)g(y)|_{x=y} \quad (2)$$

and $\theta_{\mu\nu}$ is a constant parameter characterizing the noncommutativity. Field theories on such space-time have several interesting features which are distinct from the models on the commutative space-time. These includes UV/IR mixing, novel topological soliton solutions, twisted symmetries etc. Gauge theories in NC space time can be mapped using Seiberg-Witten (SW) map [2] to gauge theories in commutative space time. In this note we study Bose-Fermi equivalence of scalar field theory in 2+1 dimensional NC space-time. Bose-Fermi transmutation is studied for the NC theories after re expressed in the commutative space-time using SW map(keeping up to order θ terms.) Bosonisation of Fermionic theories (and vice versa)have been well studied in commuative space-time. Polyakov proposed a study of Bose-Fermi equivalence in 2+1 dimension using Chern-Simon gauge action[3]. It was shown that the expectation value of the Wilson loop averaged over Chern-Simon(CS) gauge action (and for a suitable coefficient) is given by

$$\langle e^{i\oint_C A dx} \rangle_{CS} = e^{i\pi W(C)} \quad (3)$$

where $W(C)$ is the writhe of the space curve. It was also shown [3, 4] that $W(C) = \Omega(C) + (2k+1), k \in Z$. Here $\Omega(C)$, known as Polyakov spin factor, represent the solid angle subtended by the tangent to the curve C on a unit sphere. This is also related to the overlap of spin coherent states and forms the symplectic 2-form of SU(2) group manifold which is S^2 . The odd integer $(2k+1)$ has been shown to be related to statistics. The expression in the Eqn.(3) has been applied, to derive fermionic theory from scalar field coupled with $U(1)$ gauge field governed by Chern-Simon action by one of us [5, 6]. Interesting feature of this approach is that, it is non-perturbative in coupling constant. In this work we apply this approach to NC Space-time.

Bose-Fermi equivalence can be seen as duality equivalence. Duality aspects of NC field theories have been well studied [8, 9, 10, 11, 12] in recent times. There were many studies using different approaches to generalise the known duality equivalence between Maxwell-Chern-Simon theory(MCS) to Self-dual model(SD) in NC spaces. This was studied in [9, 10] using master action method but with different conclusions. By applying a dual projection procedure to NCSD model, a dual model was constructed in [11, 12] and was shown to be different from NCMCS theory. In [13], using a different approach dual of SW mapped NCMCS was obtained and shown to be different from SW mapped NC (Stückelberg compensated) SD model. These investigations showed that the duality relation present in the commutative space time need not carry forward to NC space-time. Bosonization in two and three dimensional NC space-time has been studied in [14, 15, 16]. Hence, it is interesting to investigate bosonisation in NC space time following the Polyakov approach.

In this work we study Fermionisation of $\lambda\phi^4$ theory coupled to $U_*(1)$ gauge field governed by Chern-Simon action in NC space-time. We apply SW map to re-express the theory in terms of fields in commutative space-time keeping terms up to the first order in θ , and apply the methods developed in [5, 6]. We derive the Fermionic partition function, exact in self coupling. The dual Fermionic theory obtained is nonlocal, interacting theory. We see that the Fermionic mass term does not get θ correction.

2 Scalar field in fundamental representation of $U_*(1)$

In this section, we consider Fermionisation of self interacting scalar field theory in the fundamental representation of $U_*(1)$. We start with the massive complex scalar field in fundamental representation, coupled to a Chern-Simon term in noncommutative Euclidean space described by

$$\begin{aligned}\hat{S}_\phi &= \int d^3x [(\hat{D}^\mu \hat{\phi}) * (\hat{D}_\mu \hat{\phi})^\dagger + m^2 \hat{\phi} * \hat{\phi}^\dagger - \lambda(\hat{\phi}^\dagger * \hat{\phi}) * (\hat{\phi}^\dagger * \hat{\phi}) \\ &\quad - \frac{i}{4\pi} \epsilon_{\mu\nu\lambda} (\hat{A}_\mu \partial_\nu \hat{A}_\lambda + \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda)]\end{aligned}\tag{4}$$

In the above action hatted fields are functions of non commutative (NC) co-ordinates. The covariant derivative is defined by

$$\hat{D}_\mu \hat{\phi} = \partial_\mu \hat{\phi} - i \hat{A}_\mu * \hat{\phi}.$$

Using SW map we rewrite the action in Eqn.(4) in terms of commutative fields and θ . For this we use the SW solution for $\hat{\phi}$ and \hat{A}_μ

$$\hat{A}_\mu = A_\mu - \frac{1}{2} \theta^{\alpha\beta} A_\alpha (\partial_\beta A_\mu + F_{\beta\mu}) \quad (5)$$

$$\hat{\phi} = \phi - \frac{1}{2} \theta^{\alpha\beta} A_\alpha \partial_\beta \phi. \quad (6)$$

to order θ [17]. Using this, from Eqn.(4) we get (to order θ)

$$\begin{aligned} S_\phi = & \int d^3x [D^\mu \phi (D_\mu \phi)^\dagger - y^{\mu\nu} D_\mu \phi (D_\nu \phi)^\dagger \\ & + m^2 (1 + y^\mu_\mu) \phi \phi^\dagger - \lambda (\phi \phi^\dagger)^2 (1 - \frac{1}{2} \theta^{\alpha\beta} F_{\alpha\beta}) - \frac{i}{4\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda] \end{aligned} \quad (7)$$

where

$$y^{\mu\nu} = \frac{1}{2} (\theta^{\mu\alpha} F_\alpha^\nu + \theta^{\nu\alpha} F_\alpha^\mu + \frac{1}{2} \eta^{\mu\nu} \theta^{\alpha\beta} F_{\alpha\beta}).$$

Note that we have used the fact that NC Chern-Simon term goes to commutative Chern-Simon term under SW map [18]. The coefficient of Chern-Simon terms is chosen so that dual theory is that of spin- $\frac{1}{2}$ fermion. Before integrating the scalar fields we linearise λ term in the above and re-express this term using Hubbard-Stratnovich field χ ,

$$\lambda (\phi \phi^\dagger)^2 (1 - \frac{1}{2} \theta^{\alpha\beta} F_{\alpha\beta}) = -\chi(x)^2 + 2\sqrt{\lambda} \chi(x) (\phi \phi^\dagger) (1 - \frac{1}{4} \theta^{\alpha\beta} F_{\alpha\beta}) \quad (8)$$

3 Scalar field integration

The Euclidean path integral with the above action is given by

$$Z = \int D\phi D\phi^\dagger DADBDCD\chi e^{-S_0} e^{-(\int d^3x [\frac{i}{4\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda] + i C_{\mu\nu} (y^{\mu\nu} + B^{\mu\nu}) + \chi(x)^2)} \quad (9)$$

$$S_0 = \int d^3x [D^\mu \phi (D_\mu \phi)^\dagger + B^{\mu\nu} D_\mu \phi (D_\nu \phi)^\dagger + [\tilde{m}^2 (1 - B^\mu_\mu)] \phi \phi^\dagger]$$

$C_{\mu\nu}$ and $B_{\mu\nu}$ were introduced to linearize the θ depended coupling of A field to scalar field and we use $\tilde{m}^2(x) = m^2 - 2\sqrt{\lambda} \chi(x)$.

After integrating the ϕ and ϕ^\dagger fields we get partition function as

$$Z = \int DADBDC e^{-\ln \det \mathcal{R}} e^{-i(\int C_{\mu\nu}(y_\theta^{\mu\nu} + B^{\mu\nu}) + \chi^2) + \frac{i\lambda}{4\pi^2} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda} \quad (10)$$

where the operator \mathcal{R} is given by

$$\begin{aligned} \mathcal{R} &= -(\delta^{\mu\nu} + B^{\mu\nu})D_\mu D_\nu - (D_\mu B^{\mu\nu})D_\nu + V \\ \text{and } V(x) &= [\tilde{m}^2(1 - B^\mu_\mu)]. \end{aligned} \quad (11)$$

We can use the heat kernel representation of the logarithm of determinant[19], treating \mathcal{R} as the Hamiltonian, i.e,

$$\ln \det \mathcal{R} = \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\alpha}{\alpha} \text{Tr} e^{-\alpha \mathcal{R}} \quad (12)$$

Applying the standard path integral method to this gauge invariant ‘‘Hamiltonian’’, \mathcal{R} we obtain

$$\ln \text{Det} \mathcal{R} = \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\alpha}{\alpha} \int_{x(\alpha)=x(0)}^{\infty} Dx(\tau) e^{-\int_0^\alpha d\tau [\mathcal{H} + \frac{1}{2} \dot{x}^\mu \partial^\rho B_{\rho\mu} - i \dot{x}^\mu A_\mu]} \quad (13)$$

In the above the measure $Dx(\tau) = (4\pi\epsilon)^{\frac{-3N}{2}} \prod_{i=0}^{N-1} d^3x$, and $\mathcal{H} = \frac{1}{4}(M^{\mu\nu})^{-1} \dot{x}_\mu \dot{x}_\nu + V(x(\tau))$ with $M^{\mu\nu} = (\delta^{\mu\nu} + B^{\mu\nu})$. Here we take $\epsilon \rightarrow \frac{1}{\Lambda^2}$. Also we omitted terms quadratic in $B_{\mu\nu}$ as they are of order θ^2 which can be seen by integrating C-field. After expanding the $e^{-\ln \det \mathcal{R}}$ in power series, the partition function becomes

$$Z = \int D\mathcal{A} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^n \left[\int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\alpha_i}{\alpha_i} \int_{x(0)=x(\alpha_i)}^{\infty} Dx e^{\int_0^{\alpha_i} d\tau [N(\tau_i)]} \right] e^{-\int G(x) d^3x} \quad (14)$$

Here $N(\tau_i)$ and $G(x)$ are

$$N(\tau_i) = \mathcal{H}(\tau_i) + \frac{1}{2} \dot{x}_i^\mu \partial^\rho B_{\rho\mu}(\tau_i) - i \dot{x}_i^\mu A_\mu(\tau_i) \quad \text{and}$$

$$G(x) = i C_{\mu\nu} (y^{\mu\nu} + B^{\mu\nu}) + \chi^2 + \frac{i}{4\pi} \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

respectively and the measure $D\mathcal{A} = DADBDCD\chi$.

4 Gauge field integration

We first rewrite $\int_x C_{\mu\nu} y_\theta^{\mu\nu}$ as $\int_x \Gamma_\theta^\nu A_\nu$ (after omitting surface terms) where Γ_θ^ν is

$$\Gamma_\theta^\nu = [-\theta^{\mu\alpha} \partial_\alpha C_\mu^\nu + \theta^{\mu\nu} \partial^\sigma C_{\mu\sigma} - \frac{1}{2} \theta^{\alpha\nu} \partial_\alpha C_\gamma^\gamma]. \quad (15)$$

For gauge field integration we collect all the A_μ terms in the above partition function and write them as

$$e^{-i \int d^3x [\frac{1}{4\pi} A^\mu d_{\mu\nu} A^\nu] - A_\mu(x)(-J^\mu + \Gamma_\theta^\mu)}$$

where we have used the definition for the particle current, the current associated with the particle moving along the Wilson loop, as

$$J_\mu = \int d\tau \dot{x}_\mu d\tau \delta^3(x - x^c(\tau)) \quad (16)$$

and $d_{\mu\lambda} = \epsilon_{\mu\nu\lambda} \partial^\nu$. Note that unlike in the commutative case, in the absence of Chern-Simon term, particle current is non-vanishing. After the gauge field integration (omitting θ^2 terms) the partition function become

$$Z = \int D\Omega \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^n \left[\int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\alpha_i}{\alpha_i} \int_{x(0)=x(\alpha_i)} Dxe^{-S_1} e^{-\int_0^{\alpha_i} d\tau [\omega_i]} \right]. \quad (17)$$

In the above

$$\omega_i = \mathcal{H}_i - 2\pi \Gamma_\theta^\mu (d^{-1})_{\mu\rho} J_i^\rho + L_1^i \quad \text{and the measure } D\Omega = DBDCD\chi.$$

Where we have used

$$L_1^i = i\pi (J_i^\mu (d_{\mu\nu})^{-1}) J_i^\nu \quad \text{and} \quad S_1 = \int d^3x [iC_{\mu\nu} B^{\mu\nu} + \chi^2]$$

In the above partition function the integral $e^{-\int d\tau L_1^i}$ is of the form

$$e^{-i\pi \int d^3x J_\mu (d_{\mu\nu})^{-1} J_\nu} = e^{i\pi (W(C_n) + \sum_{i \neq j} 2n_{ij})} \quad (18)$$

where $W(C_n)$ is the writhe of the curve $C_n (= \cup C_i)$ and n_{ij} is the linking number of the curves C_i and C_j [3]. The linking number term does not contribute. The writhe $W(C_n)$ has the expression $W(C_n) = \Omega(C_n) + 2k + 1$, where $\Omega(C_n)$ is the Polyakov factor [3], and $2k+1$ is an odd integer. Thus

$$e^{-\pi i W(C_n)} = (-1) e^{-i \frac{1}{2} \Omega(C_n)}. \quad (19)$$

The coefficient of $W(C_n)$ is dictated by the choice of the coefficient of Chern-Simon term. Interestingly this encodes both spin and statistics of the transformed field. The (-1) in the above equation is responsible for the expression appearing as determinant rather its inverse (see Eqn.(27) below), which leads to Grassmanian nature of the transmuted field. The coefficient $\frac{1}{2}$ of Polyakov's spin factor is responsible for the spin $\frac{1}{2}$ nature of the transmuted field through the well known properties of spin $\frac{1}{2}$ coherent states [20]. Using Eqn.(19), the partition function becomes.

$$Z = \int D\Omega e^{-\int d^3x (iC_{\mu\nu}B^{\mu\nu} + \chi^2)} e^{-\int \frac{d\alpha}{\alpha} \int Dx(\tau) e^{-\int_0^\alpha d\tau [\tilde{M}] + (-1)i\frac{1}{2}\Omega - iV_\mu J^\mu}} \quad (20)$$

where

$$\tilde{M} = \frac{1}{4}(\delta^{\mu\nu} - B^{\mu\nu})\dot{x}_\mu\dot{x}_\nu + V(\tau_i)$$

and V_μ is given by

$$V_\mu = [2\Gamma_\theta^\sigma(d_{\sigma\mu})^{-1} + \frac{i}{2}\partial^\rho B_{\rho\mu}] \quad (21)$$

The addition of Polyakov spin factor to the path integral for spinless particle both in free and in the presence of background scalar and vector fields have been studied in [7]. Following this procedure, we obtain

$$\begin{aligned} & - \int_{\Lambda^{-2}}^\infty \frac{d\alpha}{\alpha} \int Dx(\tau) e^{-\int_0^\alpha d\tau [\frac{1}{4}(\delta^{\mu\nu} - B^{\mu\nu})\dot{x}_\mu\dot{x}_\nu + V] + (-1)i\frac{1}{2}\Omega - i\oint V_\mu dx^\mu} \\ & = (-1) \int_{\Lambda^{-2}}^\infty \frac{d\alpha}{\alpha} Tr e^{-\alpha[\frac{\mathcal{D}}{\mathcal{A}} + \tilde{V} + M_F]} \end{aligned} \quad (22)$$

where Λ is cut-off. This makes use of the well known result

$$\int_{\hat{n}(0)=\hat{n}(l)} \mathcal{D}\hat{n} e^{i\int_0^l d\tau (H(\hat{n}) + \frac{1}{2}\Omega(\hat{n}))} = Tr \langle \hat{n} | e^{iH(\tau_\mu)} | \hat{n} \rangle \quad (23)$$

where \hat{n} are the $SU(2)$ coherent states and τ_μ are the Pauli matrices. Here we define

$$\mathcal{D} = (i\partial_\mu - V_\mu)\tau^\mu, \quad \mathcal{A} = \sqrt{\det(\delta_{\mu\nu} - B_{\mu\nu})}, \quad (24)$$

$$\tilde{V} = -\frac{\sqrt{\pi}}{4\Lambda} [(m^2 B_\mu^\mu + 2\sqrt{\lambda}\chi(x)(1 - B_\mu^\mu))] \quad (25)$$

$$\text{and } M_F = \frac{\sqrt{\pi}}{4\Lambda} (m^2 + \Lambda^2 \ln 2) \quad (26)$$

where M_F is mass of the Fermion. Using the above result in (20) we get

$$Z = \int DBDCD\chi e^{-\int d^3x(C_{\mu\nu}B^{\mu\nu}+\chi^2)} \det \left[\frac{\mathcal{D}}{\mathcal{A}} + \tilde{V} + M_F \right] \quad (27)$$

Note that -1 in Eqn.(22) is responsible for the determinant to appear in the numerator. This can be written as functional integral over fermionic fields and then integrating over χ , we get the partition function as

$$Z = \int DBDCD\Psi D\bar{\Psi} e^{-\int d^3x(C_{\mu\nu}B^{\mu\nu})} e^{-\int d^3x\bar{\Psi}\left[\frac{2\mathcal{D}}{\mathcal{A}}+\tilde{V}_1+M_F\right]\Psi-g(\Psi\bar{\Psi})^2} \quad (28)$$

where

$$\tilde{V}_1 = -\frac{\sqrt{\pi}}{4\Lambda}[m^2 B^\mu_\mu] \quad \text{and} \quad g(x) = \frac{\pi\lambda}{16\Lambda^2}(1 - B^\mu_\mu(x))^2.$$

This result is non perturbative in λ and the interacting fermionic theory is non-local. When $\lambda \rightarrow 0$ the theory continues to be non-local. This theory is different from the theory derived from a naive generalization of Fermionic theory one obtains in NC space-time by expanding $*$ product to first order in θ . Such theory will not have non-locality. In the limit $\theta \rightarrow 0$, Lagrangian in Eqn.(28) becomes

$$\begin{aligned} L = & \int d^3x (C_{\mu\nu}B^{\mu\nu}) + 2\bar{\Psi}\frac{1}{\mathcal{A}}[i\partial_\mu + \frac{i}{2}\partial^\rho B_{\rho\mu}]\tau^\mu\Psi \\ & - \bar{\Psi}\frac{\sqrt{\pi}}{4\Lambda}[m^2 B^\mu_\mu] + M_F\bar{\Psi}\Psi - \frac{\pi\lambda}{16\Lambda^2}(1 - B^\mu_\mu(x))^2(\bar{\Psi}\Psi)^2. \end{aligned} \quad (29)$$

Now integration over the field $C_{\mu\nu}$ (In the partition function) set $B_{\mu\nu}$ to vanish. Hence in the case of $\theta \rightarrow 0$ but $\lambda \neq 0$ the commutative result in [6], which is a local $(\Psi\bar{\Psi})^2$ fermionic theory is retrieved. When $\theta \rightarrow 0$ and $\lambda = 0$ we get the commutative result, i.e free fermion. Thus the commutative limit is smooth.

5 Conclusion

In this paper we have studied Fermionization in 3 dimensional NC space, where NC $\lambda\phi^4$ theory coupled to $U_*(1)$ gauge field governed by Chern-Simon action. The dual Fermionic partition function derived, (non-perturbative in λ) is non-local for both $\lambda = 0$ and $\lambda \neq 0$ cases. As it is clear Fermionic mass term does not get θ correction. In $\theta \rightarrow 0$ limit commutative result

[5, 6] is recovered. Note also when $m \rightarrow 0$, Dirac particle has a non zero mass (dependent on cut off Λ). This is expected as Chern-Simon term in the Bosonic theory is parity violating, which is reflected in the non zero mass of the fermionic theory. In the NC case it is possible for a real scalar to couple with gauge field (unlike in the commutative case). Thus it is natural to seek Fermionisation of real scalar coupled to Chern-Simon term. The SW mapped action for real scalars is (up to order θ)

$$S_\varphi = \frac{1}{2} \int d^4x \left[\partial^\mu \varphi \partial_\mu \varphi + 2\theta^{\mu\alpha} F_\alpha{}^\nu \left(-\partial_\mu \varphi \partial_\nu \varphi + \frac{1}{4} \eta_{\mu\nu} \partial^\rho \varphi \partial_\rho \varphi \right) \right]. \quad (30)$$

Here the coupling to gauge field is through non-minimal coupling only. For the application of Polyakov's approach it is necessary to have Wilson loop term (i.e, minimal coupling), which is absent here. Hence, straight forward extension of this procedure to real scalar is not possible. It is an interesting problem to see how the real scalar can be fermionised.

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